FIITJEE COMMON TEST

TWO YEAR CRP (CTY - 1820)

BATCH: B - LOT

PCM (PAPER – I) PHASE - I

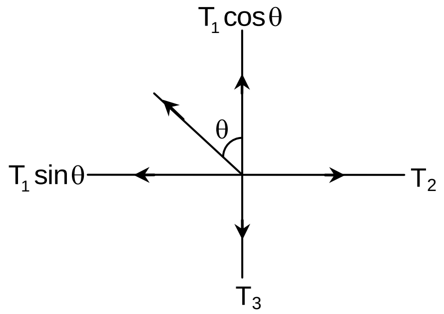
PAPER CODE: XXXX.X **DATE: DD.MM.YYYY**

HINTS AND SOLUTIONS [SET – A]

PHYSICS

PART – A

1. **B, D**
Resolve The forces on FBD



$T_1 \sin\theta = T_2$

$T_1 \cos\theta = T_3$

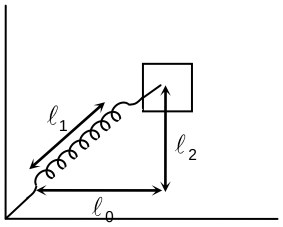
Simultaneously solve the equations.

2. **B, C, D**
Differentiate w.r.t. time to get acceleration
Then selectively use components to get radial and tangential acceleration.

3. **A, C**
Because the surface is rough friction force will be working opposite to the motion of block.

4. **A, D**
$\mu mg = \text{frictional force}$
Compare the applied force with frictional force to understand block's motion.

5. **B, C, D**



$\ell_1 = \sqrt{\ell_2^2 + \ell_0^2}$

$\ell_0 - \ell_1 =$ elongation x

$\frac{1}{2}kx^2$ = work done by spring

6. **A, D**

Subtract $\left[S_2 = ut_2 - \frac{1}{2}gt_2^2\right]$ from $\left[S_3 = ut_3 - \frac{1}{2}gt_3^2\right]$

From this distance get u the initial speed.

$u + \frac{a}{2}(2n-1) = S_n$

7. **A, C, D**

$\vec{F}.\vec{v} = 0 \Rightarrow \vec{F} \perp \vec{v}$

8. **A, B**

$\mu mg(1-\sin\theta) = mg\cos\theta$

But $mg\cos\theta$ = the force pulling the block sideway and $\mu mg(1-\sin\theta)$ is the least value for which the block would not move.

9. **A, C**

Circular motion

Velocity perpendicular to its displacement.

10. **B, C, D**

Conservation of energy and conservation of angular momentum

$I_1\omega_1 = I_2\omega_2$

$I_1 > I_2$

PART – C

1. **7**

Find the area under the curve. Add 3 (initial velocity).

2. **3**

Find the acceleration of the system of blocks, then find velocity and equate their final displacement.

3. **2**

Newton's laws of motion & constraints motion

Equate their final displacement

4. **3**

Take the component of gravity perpendicular to the slope of inclined plane.

5. **2**

The speed of the block is decelerated by the frictional force.

Then, $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

Get $\sqrt{x}$.

6. **0**

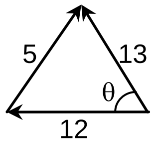
The normal force is perpendicular with respect to the displacement
Thus, work done = 0

7. **2**
$F_{net} = 02.0\ N$
$d = 1\ m$

8. **0**
$F_{net} = 0$

9. **3**
$\sqrt{\left(10\sqrt{3} - 5\sqrt{3}t\right)^2 + \left(5t\right)^2} = x$
$\frac{dx}{dt} = 0$

10. **6**



$\cos\theta = \frac{12}{13}$

CHEMISTRY

PART – A

1. **C**
2. **A, B, D**
3. **A, B, D**
4. **A, D**
5. **A, B, C**
6. **A, B, C, D**
7. **A, B, C**
8. **B, C**
9. **B, D**
10. **B, D**

PART – C

1. **5**
2. **5**
3. **4**
4. **9**
5. **6**

6. **5**

7. **4**

8. **4**

9. **1**

10. **3**

MATHEMATICS

PART – A

1. **A, C**

$y = e^{\sqrt{x}} + e^{-\sqrt{x}}$ …….. (i)

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}e^{\sqrt{x}} - \frac{1}{2\sqrt{x}}e^{-\sqrt{x}} = \frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$ …….. (ii)

$\left(e^{\sqrt{x}} - e^{-\sqrt{x}}\right)^2 = \left(e^{\sqrt{x}} + e^{-\sqrt{x}}\right)^2 - 4e^{\sqrt{x}}e^{-\sqrt{x}}$

$\left(e^{\sqrt{x}} - e^{-\sqrt{x}}\right)^2 = y^2 - 4$

$e^{\sqrt{x}} - e^{-\sqrt{x}} = \sqrt{y^2 - 4}$

$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x}}\left(\sqrt{y^2 - 4}\right)$

2. **B, D**

The director circle is $x^2 + y^2 = 50$

Now, solve this equation with the equation of given line.

3. **B, C**

Let $y = 4\sin^2 x + 3\cos^2 x - 24\sin\frac{x}{2} - 24\cos\frac{x}{2}$

$y = 3 + \sin^2 x - 24\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)$

$y = 3 + \sin^2 x - 24\sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}$

$y = 3 + \sin^2 x - 24\sqrt{1 + \sin x}$

For extreme value, put $\sin x = 0$ and $\sin x = 1$

$y_{max} = 3 - 24 = -21$

$y_{min} = 4 - 24\sqrt{2} = 4\left(1 - 6\sqrt{2}\right)$

4. **A, D**

$\sin\theta = \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{6}}{3}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

Angle made by L with positive x-axis can be

$= \frac{\pi}{4} - \frac{\pi}{6}$ and $\frac{\pi}{4} + \frac{\pi}{6}$

$= \frac{\pi}{12}$ and $\frac{5\pi}{12}$

5. **B, C**
Triangle formed by line is obtuse.

6. **A, C**
$6(\log x)^2 + \log x - 1 = 0$
$(3\log x - 1)(2\log x + 1) = 0$
$x = 10^{\frac{1}{3}}$ or $x = 10^{\frac{-1}{2}}$

7. **A, C**
Equation of circle
$(x-3)^2 + y^2 + \lambda y = 0$
$x^2 + y^2 - 6x + \lambda y + 9 = 0$
y-intercept $= 2\sqrt{f^2 - c} = 2\sqrt{\frac{\lambda^2}{4} - 9}$
$2\sqrt{\frac{\lambda^2}{4} - 9} = 2\sqrt{7} \Rightarrow \frac{\lambda^2}{4} = 16$
$\lambda^2 = 64 \Rightarrow \lambda = \pm 8$
$\therefore$ Circle is $x^2 + y^2 - 6x \pm 8y + 9 = 0$

8. **A, C**
Let point be $(3\cos\theta, 3\sin\theta)$
$\therefore \sqrt{(3\cos\theta - 1)^2 + \left(3\sin\theta - 2\sqrt{2}\right)^2} = 2$
$3\cos\theta + 6\sqrt{2}\sin\theta = 7$
Now, (A) & (C) satisfy above
$\therefore 3\cos\theta = -1, 3\sin\theta = 2\sqrt{2}$ and
$3\cos\theta = \frac{23}{9}$ & $\sin\theta = \frac{10\sqrt{2}}{9}$

9. **C**
$3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4\left(\sin^6 x + \cos^6 x\right)$
$= 3(1 - \sin 2x)^2 + 6(1 + \sin 2x) + 4\left(\sin^2 x + \cos^2 x\right)\left(\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x\right)$
$= 3\left(1 + \sin^2 2x - 2\sin 2x\right) + 6(1 + \sin 2x) + 4\left(1 - 3\sin^2 x\cos^2 x\right)$
$= 13 + 3\sin^2 2x - 3\sin^2 2x = 13$

10. **B**
$\sec^2\theta \geq 1 \Rightarrow \frac{4xy}{(x+y)^2} \geq 1 \Rightarrow (x+y)^2 \leq 4xy$
$(x-y)^2 \leq 0 \Rightarrow x - y = 0 \Rightarrow x = y$

PART – C

1. **4**
$x + y = 1, 2x + y - 1 = 0, y = 0$
Vertices of triangle (0, 1), $\left(\frac{1}{2}, 0\right)$ and (1, 0)

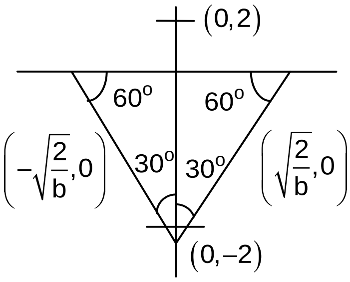
Therefore, orthocenter is $\left(0,\ \frac{-1}{2}\right)$

$\therefore k = -\frac{1}{2}$

$\therefore \frac{1}{k^2} = 4$

2. **2**

$y = bx^2 - 2$ intersection point



$\tan 30^o = \frac{\frac{\sqrt{2}}{b}}{2}$

$\frac{1}{\sqrt{3}} = \frac{1}{b\sqrt{2}}$

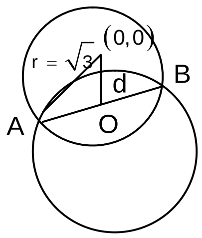
$b = \sqrt{\frac{3}{2}}$

$[b] = 1$

3. **2**

$S \equiv x^2 + y^2 - c = 0 \Rightarrow c = 3$

$S' \equiv x^2 + y^2 + ax + by + c = 0 \Rightarrow \sqrt{\frac{a^2}{4} + \frac{b^2}{4} - c} = \sqrt{6} \Rightarrow a^2 + b^2 = 36$



Equation of chord AB is $S' - S = 0$

$ax + by + 2c = 0$

$d = \left|\frac{2c}{\sqrt{a^2 + b^2}}\right| = \left|\frac{6}{6}\right| = 1$

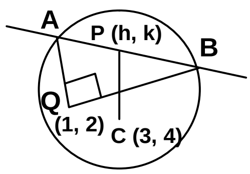
$AO = \sqrt{3 - d^2} = \sqrt{3 - 1} = \sqrt{2}$

$AB = 2AO = 2\sqrt{2} = \sqrt{8}$

$\therefore \ell = 2$

4. **8**

$PQ = PA = PB$



$\sqrt{(h-1)^2 + (k-2)^2} = \sqrt{6^2 - (h-3)^2 - (k-4)^2}$

$h^2 + k^2 - 4h - 6k - 3 = 0$

$x^2 + y^2 - 4h - 6k - 3 = 0$

$a + b + c = 2 + 3 + 3 = 8$

5. **1**

$a = \log_3 \log_3 2$

$3^{-a} = 3^{-\log_3 \log_3 2} = (\log_3 2)^{-1} = \frac{1}{\log_3 2} = \log_2 3$

$1 < 2^{-k+3^{-a}} < 2 \Rightarrow 1 < 2^{-k}.2^{3^{-a}} < 2^1$

$1 < 2^{-k}.2^{\log_2 3} < 2^1$

$1 < 2^{-k}.3 < 2 \Rightarrow \frac{1}{3} < 2^{-k} < \frac{2}{3}$

$\log_2 \frac{3}{2} < k < \log_2 3$

Hence, integral value of k is 1.

6. **3**

$\cos^2(45^o + x) + (\sin x - \cos x)^2 = \cos^2(45^o + x) + \sin^2 x + \cos^2 x - 2\sin x \cos x$

$= \cos^2(45^o + x) + 1 - \sin 2x$

For maximum value $x = -45^o$

$= \cos 0^o + 1 + \sin 90^o = 3$

7. **6**

$\text{cosec}\,10^o + \text{cosec}\,50^o - \text{cosec}\,70^o$

$= \frac{1}{\sin 10^o} + \frac{1}{\sin 50^o} - \frac{1}{\sin 70^o} = \frac{1}{\cos 80^o} + \frac{1}{\cos 40^o} - \frac{1}{\cos 20^o}$

$= \frac{\cos 40^o \cos 20^o + \cos 80^o \cos 20^o - \cos 40^o \cos 80^o}{\cos 20^o \cos 40^o \cos 80^o}$

$= \frac{\cos 20^o [2\cos 60^o \cos 20^o] - \cos 40^o \cos 80^o}{\cos 20^o \cos 40^o \cos 80^o}$

$= \frac{4(2\cos^2 20^o - 2\cos 40^o \cos 80^o)}{2 \times 4\cos 20^o \cos(60^o - 20^o)\cos(60^o + 20^o)}$

$= \frac{4[1 + \cos 40^o - \cos 120^o - \cos 40^o]}{2\cos 60^o} = 4 \times \frac{3}{2} = 6$

8. **0**

In ΔABC,

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$x + x + 1 + 1 - x = x(1 + x)(1 - x)$

$2 + x = x - x^3 \Rightarrow x^3 = -2 \Rightarrow x = (-2)^{\frac{1}{3}}$

$\tan A = x < 0 \Rightarrow A$ is obtuse

$\tan B = 1 + x \Rightarrow 1 - 2^{\frac{1}{3}} < 0 \Rightarrow$ B is obtuse

A and B are obtuse angles.

Hence, no triangle exists.

9. **2**

$\log_{\frac{1}{2}} |x - 3| > -1 \Rightarrow |x - 3| < \left(\frac{1}{2}\right)^{-1}$

$|x - 3| < 2 \Rightarrow -2 < x - 3 < 2$

$1 < x < 5 \Rightarrow x = 2,\ 4$

10. **8**

$y = 4\sin^2 x \Rightarrow \frac{dy}{dx} = 4.2\sin x \cos x$

$\frac{dy}{dx} = 4\sin 2x \Rightarrow \frac{dy}{dx} = \frac{8}{2}\sin 2x$

$\therefore n = 8$