

FIITJEE**JEE(Main)-2019****ANSWERS, HINTS & SOLUTIONS****PART TEST – I
(Main)****ALL INDIA TEST SERIES**

Q. No.	PHYSICS	Q. No.	CHEMISTRY	Q. No.	MATHEMATICS
1.	A	31.	B	61.	B
2.	A	32.	C	62.	C
3.	A	33.	B	63.	A
4.	D	34.	B	64.	B
5.	A	35.	B	65.	A
6.	A	36.	C	66.	B
7.	B	37.	D	67.	D
8.	D	38.	B	68.	B
9.	D	39.	C	69.	C
10.	C	40.	D	70.	A
11.	A	41.	B	71.	D
12.	C	42.	A	72.	C
13.	B	43.	C	73.	C
14.	D	44.	D	74.	B
15.	D	45.	B	75.	D
16.	B	46.	B	76.	B
17.	C	47.	C	77.	D
18.	B	48.	C	78.	A
19.	A	49.	B	79.	A
20.	B	50.	B	80.	D
21.	A	51.	C	81.	C
22.	C	52.	A	82.	C
23.	A	53.	D	83.	B
24.	A	54.	C	84.	B
25.	A	55.	D	85.	C
26.	C	56.	B	86.	C
27.	A	57.	D	87.	B
28.	D	58.	C	88.	B
29.	B	59.	D	89.	B
30.	C	60.	C	90.	A

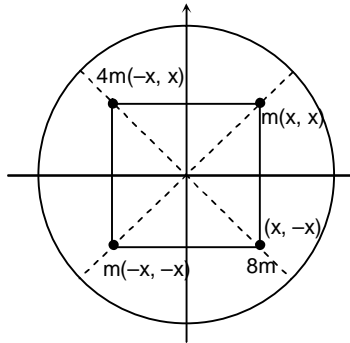
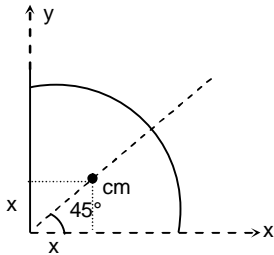
Physics

PART – I

SECTION – A

1. $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$
 $\frac{\int \vec{v}_{rm} dt}{\int dt} = \frac{\int \vec{v}_r dt}{\int dt} - \frac{\int \vec{v}_m dt}{\int dt}$
 $\langle \vec{v}_{rm} \rangle = \vec{v}_r - 0.$

2. $x_{cm} = \frac{m(x) + 4m(-x) + 5m(-x) - 8m(x)}{m + 4m + 5m + 8m} = 0$
 $y_{cm} = \frac{m(x) + 4m(x) + 5m(-x) + 8m(-x)}{18m}$
 $= -\frac{8mx}{18m} = -\frac{4}{9}x = -\frac{4}{9} \times \left(\frac{4R}{3\pi}\right) = -\frac{16R}{27\pi}$



3. Vector along the normal of plane is $-\hat{i} - \hat{j}(1-b)$
 Coefficient of \hat{k} is zero always also it becomes \hat{i} when $b = 1$.

4. $I_0 = Kmb^2$
 $I' = K \cdot (m) \cdot \left(\frac{b}{2}\right)^2$
 $I' = (Kmb^2) \frac{1}{4}$
 $\Rightarrow I' = \frac{I_0}{4}.$

5. $\frac{v\sqrt{3}}{2} = 50 \times \frac{4}{5} \Rightarrow v = \frac{80}{\sqrt{3}}$
 Now using conservation of mechanical energy
 $\frac{(80)^2}{3} - (50)^2 = -2gh$
 $\Rightarrow \frac{-6400 + 7500}{3} = 2gh$

$$\Rightarrow \frac{1100}{3 \times 10 \times 2} = h \Rightarrow h = \left(\frac{55}{3}\right) \text{m}$$

6. $v^2 + x = 10$

$$2va + v = 0$$

$$\Rightarrow a = -\frac{1}{2} \text{ms}^{-2}$$

7. $p_1 = p_2$

$$V_1 < V_2$$

$$\frac{p^2}{2m} = E$$

$$\Rightarrow E_1 < E_2$$

8. $I = k \cdot ma^2$

$$\Delta I = \frac{2kma^2 \cdot \Delta a}{a}$$

$$= \frac{2I \cdot \Delta a}{a}$$

9. Basic concept of FBD and equilibrium

10. Basic concept of pseudo force

11. $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times 10^4 \times 10^2 = 2.5 \times 10^5 \text{ J}.$

10% of this is stored in the spring.

$$\frac{1}{2}kx^2 = 2.5 \times 10^4$$

$$x = 1 \text{ m}$$

$$k = 5 \times 10^4 \text{ N/m}.$$

12. Basic concept of projectile

13. $X_{\text{cm}} = \frac{\lambda L^{n+2}}{(n+2)} \cdot \frac{n+1}{\lambda L^{n+1}} = \frac{3L}{4}.$

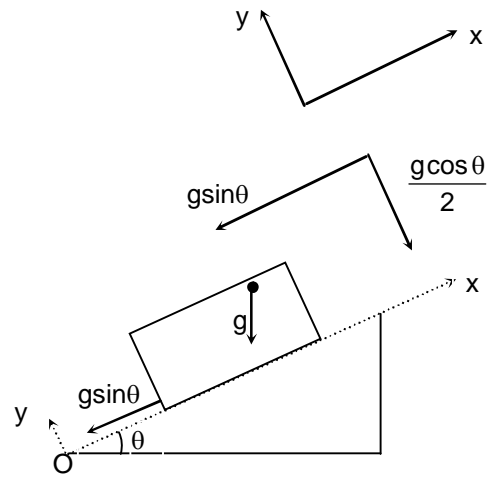
$$\frac{n+1}{n+2} = \frac{3}{4}$$

$$n = 2$$

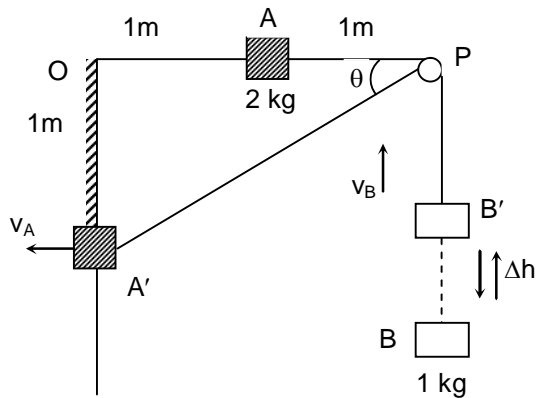
14. With equal initial kinetic energy, one can write for these rotating disks that $KE = \frac{1}{2} I \omega^2 = \frac{L^2}{2I}$. As a result $L = \sqrt{2I(KE)}$ which means $L_X < L_Y$ as object Y has more mass. By applying the same force on the outside of each disk, the torque from the centre of each disk is the same. From the angular impulse momentum theorem, $(\tau)\Delta t = \Delta L$ and since the torques and times are equal, the change in angular momentum is the same for each. Consequently, $L_X < L_Y$ after the push as well. The kinetic energy depends on the total distance through which each force acted. Since disk X is rotating at a higher rate than disk Y and the applied force will increase the angular speed of each disk, there will be a greater angle swept out by a disk X compared to disk Y. As a result, the kinetic energy change for disk X is greater than for disk Y meaning that after the push, $K_X > K_Y$.

15. $\vec{a}_{CM} = \frac{\vec{F}_{net}}{\sum m_i}$

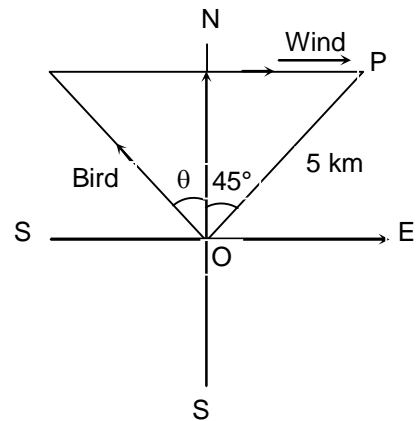
16. $\vec{a}_{Bg} = -g\sin\theta\hat{i} - \frac{g\cos\theta}{2}\hat{j} \Rightarrow$ Acceleration of box with respect to ground
 $\vec{a}_{Pg} = -g\sin\theta\hat{i} - g\cos\theta\hat{j} \Rightarrow$ Acceleration of particle with respect to ground
 $\vec{a}_{PB} = -\frac{g\cos\theta}{2}\hat{j} \Rightarrow$ Acceleration of particle with respect to Box
 $\vec{u}_{PB} = u\cos 45^\circ\hat{i} + u\sin 45^\circ\hat{j} \Rightarrow$ Initial velocity of particle with respect to Box
 Time of flight = $\frac{2u\sin 45^\circ}{\frac{g\cos\theta}{2}} = \frac{2\sqrt{2}u}{g\cos\theta}$



17. $A'P = \sqrt{2^2 + 1} = \sqrt{5}$
 $\Rightarrow \Delta h = (\sqrt{5} - 1)$
 From work energy theorem :
 $+2g \times 1 - 1 \times g\Delta h = \frac{1}{2} 2v_A^2 + \frac{1}{2} \times 1 \times v_B^2$
 Where $v_B = v_A \cos\theta$; $\cos\theta = \frac{2}{\sqrt{5}}$
 On solving :
 $7.5 = \frac{7}{5} v_A^2 \Rightarrow v_A = 2.3 \text{ m/sec.}$

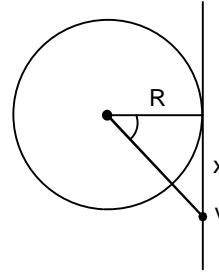


18. Bird has to fly away from wind direction as shown so as to reach P.
 $\therefore V_{Bird} > V_{wind}$
 If t is time taken;
 $\frac{5}{\sqrt{2}} \times \frac{1}{15\cos\theta} = t \dots(i)$
 $(5 - 15\sin\theta)t = \frac{5}{\sqrt{2}} \dots(ii)$



Solving for t we get
 $8t^2 + \sqrt{2}t - 1 = 0 \Rightarrow t = \left(\frac{\sqrt{34} - \sqrt{2}}{16} \right) \text{ hrs.}$

19. $\frac{x}{R} = \tan \theta$
 $x = R \tan \theta$
 $\Rightarrow \frac{dx}{dt} = R \sec^2 \theta \frac{d\theta}{dt} = R\omega \sec^2 \theta$
 $\Rightarrow v = R\omega \sec^2 \theta$
 $\Rightarrow \frac{dv}{dt} = R\omega \left[2 \sec \theta \cdot \sec \theta \tan \theta \frac{d\theta}{dt} \right]$
 $\Rightarrow a = 2R\omega^2 \sec^2 \theta \tan \theta$
 $v = R(2) \sec^2 60^\circ = 2R(4) = 8R$
 $a = 2(R)(4)(4)(\sqrt{3}) = 32\sqrt{3}R$



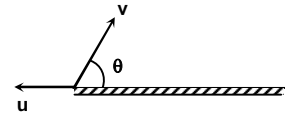
20. Let frog takes off at θ angle with v speed from ground frame

$$v_P = \frac{mv \cos \theta}{3m} = \frac{v \cos \theta}{3}$$

$$\Rightarrow (\vec{v}_{t/P})_x = \vec{v}_t - \vec{v}_P = v \cos \theta + \frac{v \cos \theta}{3} = \frac{4}{3} v \cos \theta$$

$$(\vec{v}_{t/P})_y = v \sin \theta \Rightarrow R_{t/P} = \frac{2 \left(\frac{4}{3} \right) v \cos \theta v \sin \theta}{g} = \ell$$

$$\Rightarrow v^2 \sin 2\theta = \frac{3}{4} g \ell \Rightarrow v_{\min} = \sqrt{\frac{3}{4} g \ell}$$



21. $\hat{v}_i = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}$
 $\hat{p} = \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{7}$
 $\hat{v}_f = \vec{v}_i - 2(\hat{v}_i \cdot \hat{p})\hat{p}$
 $= \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{4+9+16}} - \frac{2(6+18+8)(3\hat{i} - 6\hat{j} + 2\hat{k})}{\sqrt{29} \times 49}$
 $= \frac{-94\hat{i} + 237\hat{j} + 68\hat{k}}{49\sqrt{29}}$

22. Time of flight = $\frac{2 \times 10}{10} = 2s$;
 Range = $2 \times 15 = 30$ m

23. Area = $\frac{1}{2} \times 0.10 \times 120 = 6$ Ns
 Change in momentum = $6 - 400 \times 10^{-3} \times 10 \times 0.10$ (Due to Gravitation) = $6 - 0.4 = 5.6$ Ns
 $mv = 5.6 \Rightarrow v = 14$ m/s
 $H = \frac{v^2}{2g} = 9.8$ m

24. $\omega_r = v_0 R$, and $\omega_z = \frac{v_0}{O'O}$

$OO' = R \cot \alpha$

$\vec{\omega} = [\omega_z \hat{k} + \omega_r (-\hat{i})]$

$\therefore \vec{v}_A = \vec{\omega} \times \vec{r}_A = \vec{\omega} \times [R \cot \alpha (\hat{i}) + R(\hat{k})]$

$= \left[\frac{v_0}{R \cot \alpha} \hat{k} - \frac{v_0}{R} \hat{i} \right] \times [R \cot \alpha \hat{i} + R \hat{k}]$

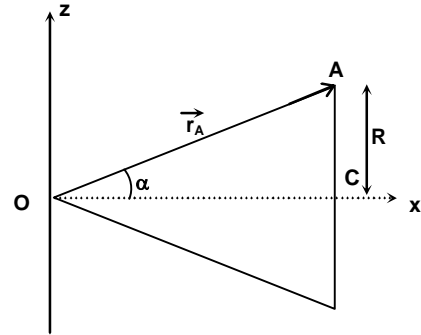
$\Rightarrow \vec{v}_A = (v_0 + v_0) \hat{i} = 2v_0 \hat{i} \Rightarrow v_A = 2v_0$

Similarly, $\vec{v}_B = \vec{\omega} \times \vec{r}_B$

Where, $\vec{r}_B = R \cot \alpha \hat{i} - R \hat{j}$

$\Rightarrow \vec{v}_B = -v_0 \hat{k} + \frac{v_0}{\cot \alpha} \hat{i} + v_0 \hat{j}$

$\Rightarrow v_B = \sqrt{v_0^2 + \frac{v_0^2}{\cot^2 \alpha} + v_0^2} = v_0 \sqrt{2 + \tan^2 \alpha}$



25. $a_1 = \frac{F_0}{m} \sin \omega t = \left(\frac{F_0}{m \omega^2} \right) \omega^2 \sin \omega t = \omega^2 R \sin \omega t$

$a_2 = \frac{F_0}{m} \cos \omega t = \left(\frac{F_0}{m \omega^2} \right) \omega^2 \cos \omega t = \omega^2 R \cos \omega t$

Here $R = \frac{F_0}{m \omega^2}$

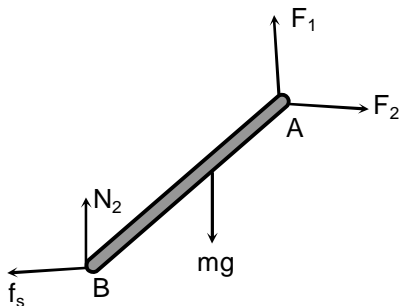
$v_1 = \frac{F_0}{m \omega} (1 - \cos \omega t)$ & $v_2 = \frac{F_0}{m \omega} \sin \omega t \Rightarrow x_1 = R(\omega t - \sin \omega t)$ & $y_2 = R(1 - \cos \omega t)$

$\langle v_1 \rangle = \frac{F_0}{m \omega}$, and $\langle v_2 \rangle = 0$

26. No external force is acting along horizontal direction

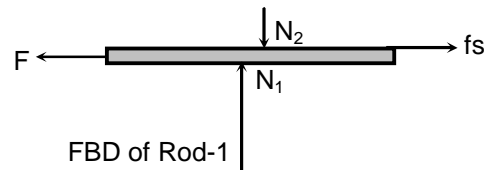
27. $E = U + K$, for a given E, k will be maximum where U will be minimum.

28. Taking torque about A $mg \frac{l}{2} \cos \alpha = f_s l \sin \alpha + N_2 l \cos \alpha$



FBD of Rod-2

$\Rightarrow N_2 = \frac{mg}{2(1 + \mu \tan \alpha)}$

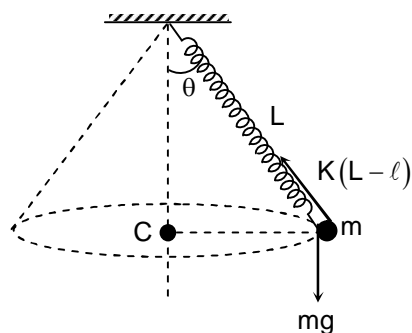


FBD of Rod-1

$$F \geq f_s \Rightarrow F_{\min} = f_{s\max} = \mu N_2 \Rightarrow F_{\min} = F_2 = \frac{\mu mg}{2(1 + \mu \tan \alpha)} = \frac{0.5 \times 3 \times 10}{2(1 + 0.5 \times 1)} = 5 \text{ N}$$

29.

$$\begin{aligned} mg &= K(2\ell - \ell) = K\ell \\ K(L - \ell) \cos \theta &= mg \\ \frac{mg}{\ell}(L - \ell) \frac{\sqrt{L^2 - r^2}}{L} &= mg \\ \Rightarrow \sqrt{L^2 - r^2} &= \frac{L\ell}{L - \ell} \\ \Rightarrow L^2 - \left(\frac{L\ell}{L - \ell}\right)^2 &= r^2 \\ \Rightarrow L^2 \left(1 - \frac{\ell^2}{(L - \ell)^2}\right) &= r^2 \\ \Rightarrow r^2 &= L^2 \left(\frac{L^2 - 2L\ell}{(L - \ell)^2}\right) \\ \Rightarrow r &= \frac{L}{\sqrt{L - \ell}} \sqrt{L(L - 2\ell)} \end{aligned}$$



$$30. \quad \frac{mv^2}{R} = mg \cos \theta \quad \dots(1)$$

$$\frac{1}{2}(M + m)v^2 = mg(1 - \cos \theta)R + \frac{\pi R}{4}Mg \quad \dots(2)$$

$$\frac{1}{2}(M + m)g \cos \theta = mg(1 - \cos \theta) + \frac{\pi Mg}{4}$$

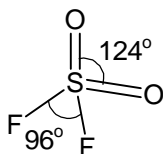
$$\Rightarrow (M + m) \cos \theta = 2m(1 - \cos \theta) + \frac{\pi}{2}M$$

Chemistry

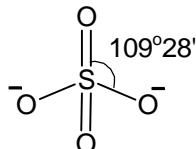
PART - II

SECTION - A

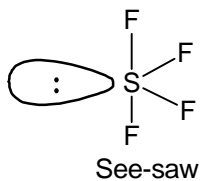
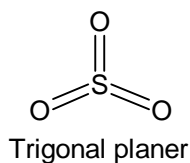
31.



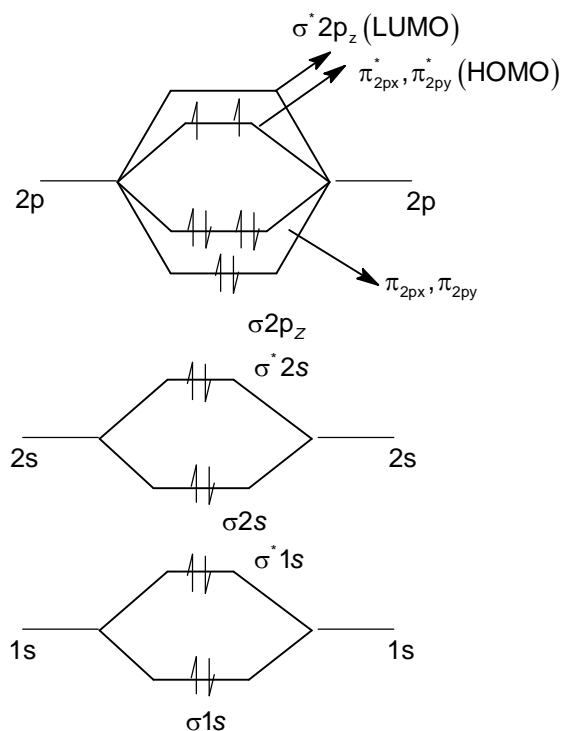
sp^3 but not ideal tetrahedral.



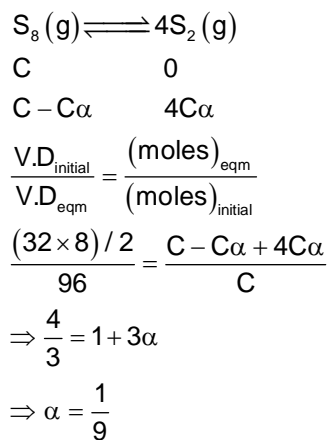
Ideal tetrahedral



33.



35.



$$\% \text{ decomposition} = \frac{1}{9} \times 100 = 11.11\%$$

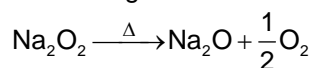
36.

- 'Hf' form interstitial hydride, e.g. HfH_{1.98}
 → KH is an ionic hydride
 → SiH₄ is a covalent hydride
 → 'Co' don't form hydride (hydride gap)

37.

- Actual boiling point order
 H₂S < H₂Se < H₂Te < H₂O

38. On heating



$\Rightarrow \text{Na}_2\text{O}_2 \Rightarrow \text{O}_2^{2-} \Rightarrow$ it is diamagnetic in nature.

$\Rightarrow \text{Na}_2\text{O}_2 + \text{CO}_2 \longrightarrow \text{Na}_2\text{CO}_3$ (Suitable for use in air purification)

$\Rightarrow \text{Na}_2\text{O}_2$ is good oxidizing agent.

41. $\text{S}^{+4} \longrightarrow \text{S}^{+6} + 2\text{e}^-$ (Oxidation half reaction)

$\text{Cr}^{+6} + 3\text{e}^- \longrightarrow \text{Cr}^{+3}$ (Reduction half reaction)

$\therefore \text{Na}_2\text{CrO}_4$ is oxidizing agent

\therefore Eq. wt. of Na_2CrO_4 (oxidizing agent) = $M/3$

42. $2\text{CH}_4(\text{g}) + 4\text{H}_2\text{O}(\text{g}) \rightleftharpoons 2\text{CO}_2(\text{g}) + 8\text{H}_2(\text{g})$... (i)

$\text{CO}(\text{g}) + \text{H}_2\text{O}(\text{g}) \rightleftharpoons \text{CO}_2(\text{g}) + \text{H}_2(\text{g})$... (ii)

Equation (i) - 2 × Equation (ii)

$2\text{CH}_4(\text{g}) + 2\text{H}_2\text{O}(\text{g}) \rightleftharpoons 2\text{CO}(\text{g}) + 6\text{H}_2(\text{g})$... (iii)

Equilibrium constant for Equation (iii) = $\frac{K_1}{K_2^2}$

By multiplying Eq. (iii) by $\frac{1}{2}$ we get

$\text{CH}_4(\text{g}) + \text{H}_2\text{O}(\text{g}) \rightleftharpoons \text{CO}(\text{g}) + 3\text{H}_2(\text{g})$

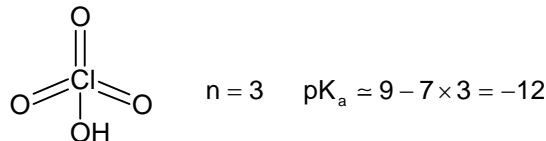
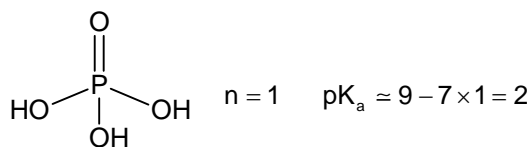
\therefore Equilibrium constant = $\left(\frac{K_1}{K_2^2}\right)^{1/2} = \frac{\sqrt{K_1}}{K_2}$

43. Possible value of quantum number of 3d orbital.

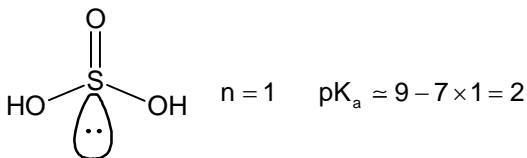
$n = 3, \ell = 2, m = -2, -1, 0, +1, +2$

$s = +1/2, -1/2$

44.

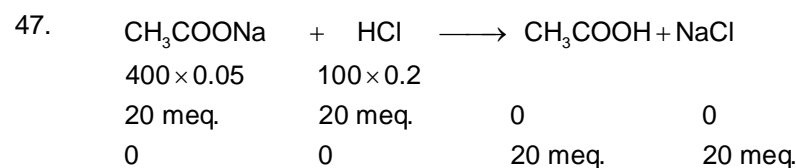
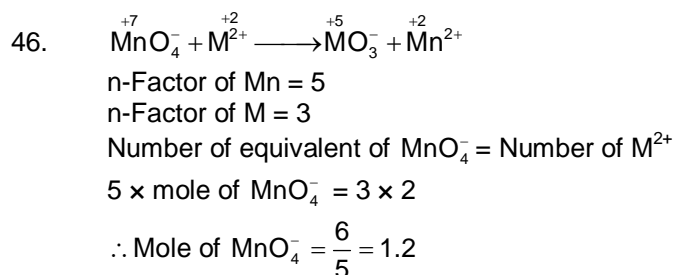


$\text{H}-\text{O}-\text{Cl} \quad n = 0 \quad \text{p}K_a \approx 9 - 7 \times 0 = 9$



45.

	n		P
D	1		1
O ¹⁸	10		8
H	0		1
x = 11			y = 10
x - y = 1			



$$\therefore [\text{CH}_3\text{COOH}] = \frac{20}{500} = 0.04 \text{ M}$$

$$\text{pH} = \frac{1}{2}(\text{pK}_a - \log C) = \frac{1}{2}(4.75 - \log(0.04))$$

$$= \frac{1}{2}\left(4.75 - \log\left(\frac{1}{25}\right)\right)$$

$$= \frac{1}{2}(4.75 + \log(25))$$

$$= \frac{1}{2}(4.75 + 1.4) = 3.075$$

48. Uncertainty in velocity

$$\Delta v = \frac{0.2 \times 200}{100} = 0.4 \text{ ms}^{-1}$$

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x \geq \frac{h}{4 \times m \times \pi \times \Delta v}$$

$$= \frac{6.4 \times 10^{-34}}{4 \times 50 \times 10^{-3} \times 0.4 \times 3.2}$$

$$= \frac{10^{-34}}{40 \times 10^{-3}} = 2.5 \times 10^{-33}$$

49.

$$k = \frac{2.303}{t} \log\left(\frac{A_0}{A_t}\right)$$

$$\log\left(\frac{A_0}{A_t}\right) = \frac{k \cdot t}{2.303} = \frac{4.606 \times 10^{-5} \times 4 \times 3600}{2.303}$$

$$= 2 \times 4 \times 3600 \times 10^{-5}$$

$$= 0.288$$

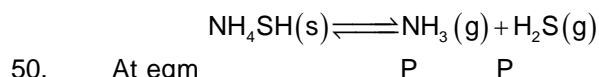
$$\left(\frac{A_0}{A_t}\right) = \text{Anti log}(2.88)$$

$$= 1.94$$

$$\text{Fraction remain} = \frac{A_t}{A_0} = \frac{1}{1.94}$$

$$\text{Fraction decomposed} = 1 - \frac{1}{1.94}$$

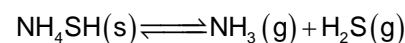
$$0.4845 \approx .48$$



$$P_{\text{total}} = 2P$$

$$K_p = P^2$$

When H_2S is added



$$(P'_{\text{NH}_3}) \quad (P'_{\text{H}_2\text{S}})$$

According to question

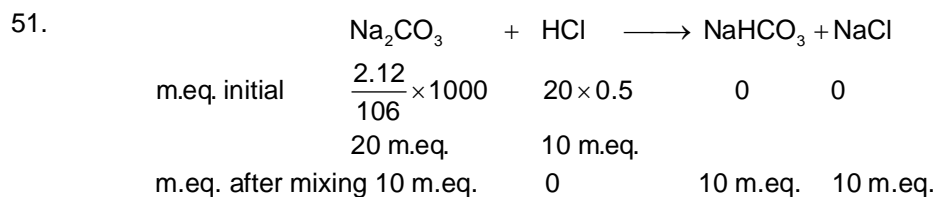
$$P'_{\text{H}_2\text{S}} = \frac{3}{2} \times P_{\text{total}} = \frac{3}{2} \times 2P = 3P$$

$$K_p = P'_{\text{H}_2\text{S}} \cdot P'_{\text{NH}_3}$$

$$P^2 = P'_{\text{NH}_3} \cdot 3P$$

$$\therefore P'_{\text{NH}_3} = \frac{P}{3}$$

$$\therefore \frac{P'_{\text{NH}_3}}{P_{\text{NH}_3}} = \frac{\frac{P}{3}}{P} = 1:3$$



Now,

Mixture contains Na_2CO_3 and NaHCO_3 so it will act as buffer.

$$\therefore \text{pH} = \text{pK}_a + \log \frac{[\text{Na}_2\text{CO}_3]}{[\text{NaHCO}_3]}$$

$$= 10.25 + \log 10/10 = 10.25$$

52. Moles of CaSO_4 in 200 ml solution = $\frac{0.34}{136} = 0.25 \times 10^{-2}$

$$\therefore \text{Molarity} = \frac{0.25 \times 10^{-2}}{200} \times 1000$$

$$= 1.25 \times 10^{-2} \text{ mol/lit.}$$

$$\begin{aligned}
 K_{sp} &= [\text{Ca}^{2+}][\text{SO}_4^{2-}] \\
 &= (1.25 \times 10^{-2})^2 \\
 &= 1.5625 \times 10^{-4}
 \end{aligned}$$

53. $t_{2/3}rd - t_{1/3}rd = \frac{2.303}{K} \log\left(\frac{A_0}{1/3A_0}\right) - \frac{2.303}{K} \log\frac{A_0}{\frac{2}{3}A_0}$

$$= \frac{2.303}{K} \log 2$$

$\Rightarrow 100 \text{ min} = \frac{0.693}{K} \Rightarrow \text{half life}$

\therefore Time taken for completion of 75% ($2t_{1/2}$) = $2 \times 100 = 200 \text{ min}$

54. $\text{Cr}^{+x} \qquad \qquad \qquad \mu = 3.87$

$$\sqrt{n(n+2)} = 3.87$$

$\Rightarrow \therefore n = 3$ (unpaired e^-)

$\therefore \text{Cr}^{+3} \Rightarrow x = 3 \qquad \qquad \qquad \therefore y = 4 (\text{Mn}^{+4})$

$$\sqrt{n(n+2)} = 3.87$$

$\Rightarrow \therefore n = 3$ (unpaired e^-)

$\therefore \text{Mn}^{+4}$

$\therefore x = 3 \quad y = 4$

$x - y = -1$

55. Rate of disappearance of 'A' = $-\frac{\Delta[A]}{\Delta t} = \frac{4 \times 10^{-2}}{40} = 10^{-3} \text{ mol L}^{-1} \text{ s}^{-1}$

We know

$$-\frac{1}{2} \frac{\Delta[A]}{\Delta t} = \frac{1}{2} \frac{\Delta[B]}{\Delta t} = \frac{1}{4} \frac{\Delta[C]}{\Delta t}$$

\therefore Rate of appearance of 'C'

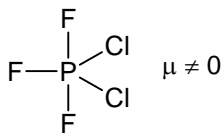
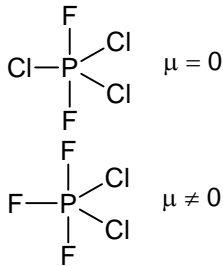
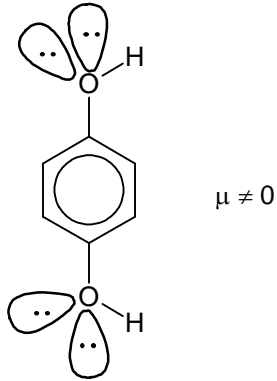
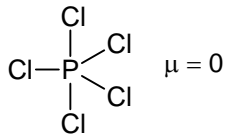
$$\begin{aligned}
 \Rightarrow \frac{\Delta[C]}{\Delta t} &= -2 \frac{\Delta[A]}{\Delta t} \\
 &= 2 \times 10^{-3} \text{ mol L}^{-1} \text{ s}^{-1}
 \end{aligned}$$

56. Here $\Delta n = -ve$

\therefore By adding inert gas backward reaction favoured

\Rightarrow By adding D(g) backward reaction takes place.

57.



58. Mass of ${}^2_1\text{H}$ = Mass of ${}^1_1\text{H}$ + mass of neutron
 = 1.0086 + 1.0078
 = 2.0164 amu
 Actual mass = 2.0064 amu
 \therefore mass defect = 2.0164 – 2.0064
 = 0.01 amu.
 \therefore Bond Energy = 0.01 \times 931.5 Mev
 = 9.315 Mev.

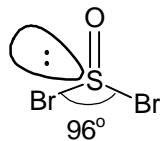
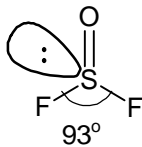
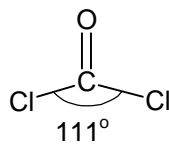
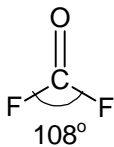
59.

Resistivity
 Standard molar entropy
 Density

Diamond
 10^{11}
 2.377
 3.513

Graphite
 1.375×10^{-5}
 5.740
 2.260

60.



Mathematics

PART – III

SECTION – A

61. If $\lim_{x \rightarrow a} f(x) = \ell$ and $\lim_{x \rightarrow a} g(x) = m \Rightarrow \lim_{x \rightarrow a} f(x) \cdot g(x) = \ell m$
62. Volume = $\pi \cdot 3^2 \cdot 2 \cdot 3 = 54\pi$
63. $(p \wedge q) \Leftrightarrow (r \wedge q)$ is equivalent to $[\sim(p \wedge q) \vee (r \wedge q)] \wedge [\sim(r \wedge q) \vee (p \wedge q)]$
64. $y = x \Rightarrow f(x + x^2) + 1 + 2x = f(x) + f(x^2) + 2x f(x)$
 $y = -x \Rightarrow f(x + x^2) + 1 + 2x = f(x) + f(x^2) + 2x f(-x)$
 $\Rightarrow f(x) = f(-x)$
65.
$$\int \frac{x^{18} - 1 \, dx}{x^7 (x^{12} + 3 + 2x^{-6})^{1/6}} = \frac{1}{12} \int \frac{12x^{11} - 12x^{-7} \, dx}{(x^{12} + 3 + 2x^{-6})^{1/6}}$$

$$= \frac{1}{12} \frac{(x^{12} + 3 + 2x^{-6})^{5/6}}{5/6} + c = \frac{(x^{18} + 3x^6 + 2)^{5/6}}{10x^5} + c$$
 $\Rightarrow P(x) = x^{18} + 3x^6 + 2$
66. $f'(x) = \lim_{h \rightarrow 0} \frac{f(x)e^h + f(h)e^x + 2xhe^x e^h - f(x)}{h}$
 $\Rightarrow f'(x) = f(x) + 2xe^x \Rightarrow f(x) = x^2 e^x$
67. One of them has to be maxima and other minima
68. There are only 2 points where $f(x)$ has local minima
69. Area = $\int_0^{x_1} \tan x - x^{1000} \, dx + \int_{x_1}^{x_2} x^{1000} - \tan x \, dx$
70. Circle is circumcircle of the triangle
71. $a_{n+1} = a_n + a_{n-1}$
 $\Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = 1 + \lim_{n \rightarrow \infty} \frac{a_{n-1}}{a_n}$
 Let $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \ell$
 $\Rightarrow \ell^2 - \ell - 1 = 0$
 $\Rightarrow \ell = \frac{1 + \sqrt{5}}{2}$
72. $f^{-1}(x) = g(x) \Rightarrow h(x) = x$
73. $3x^4 \frac{(2xy \, dx - x^2 \, dy)}{y^2} = 2y^2 \frac{(x \, dy + y \, dx)}{xy} \Rightarrow 3 \left(\frac{x^2}{y} \right)^2 d \left(\frac{x^2}{y} \right) = 2 \frac{d(xy)}{xy}$

74. Apply chain rule

75. $f(x)$ is constant function

76. $\cot \alpha > 0$

77. $\frac{dy}{dx} = \pm 1 \Rightarrow y + x = 0, y - x = 0$

78. $\frac{d^2y}{dx^2} = 4$

79. Substitute $x = r \sec \theta, y = r \tan \theta$
 $\Rightarrow \frac{rdr}{r^2 \sec \theta d\theta} = \frac{1}{2} \cos(\sin \theta) \cos^2 \theta$
 $\Rightarrow \ln|r| = \frac{1}{2} \sin(\sin \theta) + \frac{\ln c}{2}$
 $\Rightarrow \ln|x^2 - y^2| = \sin\left(\frac{y}{x}\right) + \ln c$

80.
$$\lim_{m \rightarrow \infty} \sum_{n=1}^m \left(\int_{\pi/6}^{\pi/2} f^n(x) + \int_{(3/\pi)^n}^{(2/\pi)^n} (f^{-1}(x))^{1/n} dx \right)$$

$$= \lim_{m \rightarrow \infty} \sum_{n=1}^m \left(\frac{\pi}{2} \cdot \left(\frac{2}{\pi}\right)^n - \frac{\pi}{6} \left(\frac{3}{\pi}\right)^n \right) = \frac{\frac{\pi}{2} \cdot \frac{2}{1 - \frac{2}{\pi}} - \frac{\pi}{6} \cdot \frac{3}{1 - \frac{3}{\pi}}}{\pi} = \frac{\pi}{\pi - 2} - \frac{\pi}{2\pi - 6}$$

81. $h(x)$ is a constant function and is always periodic

82.
$$\lim_{x \rightarrow 0} \left(\frac{a \frac{\sin 2x}{2x} - x^x \cdot \frac{x}{2x}}{\frac{\ln(1+2x)}{2x}} \right)^{\frac{2}{x+1}} = \frac{9}{4}$$

$$\left(\frac{a - \frac{1}{2}}{1} \right)^2 = \frac{9}{4} \Rightarrow a = 2$$

83. $\int e^{x^2+x} (4x^3 + 4x^2 + 5x + 1) dx = \int e^{x^2+x} ((2x+1)(2x^2+x) + 4x+1) dx = e^{x^2+x} (2x^2+x) + c$

84. For $\lim_{x \rightarrow \infty} f(x)$ to exist, $\lim_{x \rightarrow \infty} f'(x)$ must be 0

85.
$$\lim_{x \rightarrow \infty} \frac{\sum_{r=1}^{2018} \left(1 + \frac{r}{x}\right)^{2019}}{\prod_{r=1}^{2019} \left(1 + \frac{r}{x}\right)} = 2018$$

86. $x = 2020\pi, 2\pi$ are points of non-differentiability

87. $x = \frac{1}{e}$ is point of global minima

$$\Rightarrow 1 > \frac{1}{b} > \frac{1}{e} \Rightarrow b = 2$$

88. Let $I = \int_0^{\infty} \frac{\tan^{-1} x}{(x+1)^2} dx$

$$\text{Let } x = \frac{1}{t} \Rightarrow I = \int_0^{\infty} \frac{\cot^{-1} t}{(t+1)^2} dt$$

$$\Rightarrow I = \frac{1}{2} \frac{\pi}{2} \int_0^{\infty} \frac{1}{(x+1)^2} dx = \frac{\pi}{4}$$

89. If $f(x)$ is differentiable, then $f'(x)$ should have a unique value at that point

90. If $n(P \cup M \cup C) = 100 \Rightarrow n(M' \cap C') \leq 0 \Rightarrow n(M' \cap C') = 0$